DOCUMENT RESUME

ED 355 095 SE 053 269

AUTHOR Hojnacki, Susan K.; Grover, Barbara W.

TITLE Thinking Mathematics: What's in It for the Students?

SPONS AGENCY National Science Foundation, Washington, D.C.; Office of Educational Research and Improvement (ED), Washington, DC.

PUB DATE Apr 92

NOTE 40p.; Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA, April 21, 1992).

PUB TYPE Reports - Evaluative/Feasibility (142) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.

DESCRIPTORS Attitude Change; Educational Change; Elementary Education; *Inservice Teacher Education; Instructional Improvement; *Instructional Innovation; Mathematics Achievement; Mathematics Education; *Mathematics Instruction; Problem Solving; *Program Evaluation; Student Attitudes; Surveys; Teacher Behavior; Teacher Education Programs; *Teaching Methods; Thinking Skills

IDENTIFIERS Error Analysis (Mathematics); *Thinking Mathematics Project

ABSTRACT

The Thinking Mathematics project, a joint effort of the American Federation of Teachers and the Learning Research and Development Center, had the primary objective of developing more efficient means of disseminating new knowledge about mathematics learning and instruction. The resulting practitioner-researcher collaboration developed an instructional approach based on current research findings interpreted by the clinical wisdom of classroom teachers. This approach, called Thinking Mathematics, formed the basis of an inservice training program implemented throughout the AFT's Educational Research & Dissemination network. Teacher change--and teacher conviction in the changes being advocated--was viewed to be critical for successful student learning outcomes. Pilot implementation of Thinking Mathematics occurred at five different sites across the country during the 1990-91 academic year, involving about 65 classes from grades K-5. Cognitive and affective effects upon students involved in the year-long implementation of Thinking Mathematics were assessed by using multiple sources of data: teacher self-reports of student change, standardized achievement test scores, student attitude survey findings, and problem solving test results. The teacher self-report data show that the teachers involved in the program perceived empowering changes in their students. Project students performed as well as or better than their non-project peers on both the Computational and Concepts and Applications subsections of standardized achievement tests. Student problem solving abilities improved, as measured by the Wood-Cobb problem solving tests. Positive student attitudes towards mathematics were revealed by the project's attitude survey: students' relatively lower motivation scale scores, however, suggest that they might not yet be ready to match their efforts with their generally positive regard for mathematics. Although the results reported should be considered preliminary, there are multiple indications that student learning and attitudes were enhanced by their participation in the Thinking Mathematics program. (Contains 43 references.) (Author)

ED355095

Thinking Mathematics: What's in it for the Students?

Susan K. Hojnacki & Barbara W. Grover

Learning Research & Development Center
University of Pittsburgh
Pittsburgh, PA 15260

April 1992

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Susan K. Hojnacki

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA, on April 21, 1992, as part of a symposium entitled *Thinking Mathematics: Fruits of a Four-Year Teacher-Researcher Collaboration*. This work was supported in part by a grant from the National Science Foundation (NSF), and in part by a grant to the University of Pittsburgh's Center for Student Learning, from the Office of Educational Research and Improvement (OERI), United States Department of Education. The views expressed in this paper are not necessarily the views of NSF or OERI. We gratefully acknowledge the help of Joyce Fienberg, Marjorie Henningsen, and Mary Kay Stein. Correspondence concerning this paper should be addressed to Susan K. Hojnacki, 728 LRDC, University of Pittsburgh, Pittsburgh, PA, 15260.

SE 053 269

BEST COPY AVAILABLE

Abstract

The Thinking Mathematics project, a joint effort of the American Federation of Teachers and the Learning Research and Development Center, had the primary objective of developing more efficient means of disseminating new knowledge about mathematics learning and instruction. The resulting practitioner-researcher collaboration developed an instructional approach based on current research findings interpreted by the clinical wisdom of classroom teachers. This approach, called Thinking Mathematics, formed the basis of an inservice training program implemented throughout the AFT's Educational Research & Dissemination network. Teacher change -- and teacher conviction in the changes being advocated -- was viewed to be critical for successful student learning outcomes. Pilot implementation of Thinking Mathematics occurred at five different sites across the country during the 1990-91 academic year, involving about 65 classes from grades K-5.

Cognitive and affective effects upon students involved in the year-long implementation of Thinking Mathematics were assessed by using multiple sources of data: teacher self-reports of student change, standardized achievement test scores, student attitude survey findings, and problem solving test results. The teacher self-report data show that the teachers involved in the program perceived empowering changes in their students. Project students performed as well as or better than their non-project peers on both the Computational and Concepts and Applications subsections of standardized achievement tests. Student problem solving abilities improved, as measured by the Wood-Cobb problem solving tests. Positive student attitudes towards mathematics were revealed by the project's attitude survey; students' relatively lower motivation scale scores, however, suggest that they might not yet be ready to match their efforts with their generally positive regard for mathematics. Although the results reported should be considered preliminary, there are multiple indications that student learning and attitudes were enhanced by their participation in the Thinking Mathematics program.

Thinking Mathematics: What's in it for the Students?

Does empowering teachers empower students, and if so, how? There is widespread agreement among educational reformers that the answer ought to be "Yes" (Billups & Rauth, 1992; Lieberman, 1986; Lieberman & Miller, 1984). Most reformers also agree that implementing the broad changes in mathematics education that are being called for in numerous national reports (e.g., National Board for Professional Teaching Standards, 1989; National Council of Teachers of Mathematics, 1989, 1991; National Research Council, 1989; Mathematical Science Education Board, 1991) is both absolutely necessary and extremely difficult.

The purpose of this paper is to report preliminary findings about the extent of the benefits to students when teachers did engage in one such reform effort, the Thinking Mathematics (TM) program. Thinking Mathematics is a collaborative project between the American Federation of Teachers (AFT) and the Learning Research and Development Center (LRDC) at the University of Pittsburgh. The goal of the project was to develop more effective ways of disseminating new knowledge about mathematics instruction and learning. A group of teachers in collaboration with researchers synthesized material to be used in the AFT's Educational Research and Dissemination network.[1] This inservice training is designed to assist teachers in changing their instructional practice based on the research findings (i.e., implementation of the Thinking Mathematics program).[2]

The present paper examines the ways in which this particular teacher-researcher collaboration impacted upon the teachers and students. Data to address these issues were collected from multiple perspectives and sources. First, we set a context by discussing some of the issues related to

---

[1] The ER&D network, begun in 1981, provides a mechanism by which research studies are synthesized into language meaningful to teachers and used in professional development sessions. The process is designed to bring research findings to classroom teachers. Details about this program can be obtained from Lovely Billups, Director of Field Services, American Federation of Teachers, 555 New Jersey Avenue NW, Washington, DC 20001.

[2] A brief description of the Thinking Mathematics project is provided in Appendix A. More extensive discussions are provided in Leinhardt and Grover (1990) and Bickel and Hattrup (1991).

making teachers key figures in the change process, and how this particular collaboration addressed those issues. We conclude with a summary of the affective and cognitive effects on students in the Thinking Mathematics program.

Teacher Change as a Context for Student Change

Numerous reasons have been cited in explaining the difficulty of making lasting change (Cohen, 1987; Cuban, 1990; Passow, 1986). Teachers have been asked to commit to radical reform programs when they do not have sufficient training either in the mathematics or in the program. Cohen and Ball frame the issue of concern as follows: "How can teachers teach a mathematics that they never learned, in ways that they never experienced? That is the dilemma that such reforms pose" (1990, p. 353). Teachers are asked to commit to reform without sufficient support from systems that, in fact, are set up to punish risk-taking behaviors more often than to reward them (e.g., Silberman, 1970; Smith, 1991). In addition, there exists a lack of teacher conviction about the validity of the reform program (e.g., McLaughlin, 1990), which is exacerbated by the lack of training and support. Approaches designed to create change without addressing these insufficiencies cannot empower teachers.

Since the mid-1980s, there has been an increased awareness of the need for the professional involvement of teachers in the decision making that influences their working conditions. Romberg (1986), in reviewing what had been learned from previous experiments in mathematics curriculum change, advised that the key to successful mathematics innovation is for practitioners to be full partners in developing those changes. Teachers are called upon to take control of their own profession, gain power to influence the educational environment, and become a force in the restructuring of schools (Shanker, 1985).

Several programs in which teachers play a major role in successful change efforts are described in Lieberman's 1986 book, Rethinking school improvement: Research, craft, and concept. For example, the Interactive Research and Development on Schooling (IR&DS) program (Jacullo-Noto, 1986) involved teachers and researchers in a collaborative research and

development process that empowered teachers by "provid[ing] recognition, reinforcement, and respect for these professionals" (p. 188). The Scarsdale Teachers Institute (STI) offers staff development activities that have been developed by and are often conducted by teachers, with the support and involvement of the Scarsdale Teachers Association, the administration, the board of education, and the community. "The autonomy teachers feel through the institute structure and their freedom to create professional programs provide the impetus for their support. Teachers manage their own growth. They run the courses, hire the consultants, manage the budgets" (Schwartz, 1986, p. 204). In the mid 1980s, both the state of Connecticut and the Stanford University's Teacher Assessment Project (TAP) implemented collaborative approaches to developing alternative assessments for teachers involving public school teachers, university educators, educational researchers, and state department of education staff members (Grover, 1991). Vermont, Maine, Massachusetts, and California have recently initiated collaborative efforts in developing student assessments (Stenmark, 1991).

These efforts at empowerment, however, are not the norm. Educational reformers have generally failed to address the critical issue of teacher belief in the validity of the reform programs. Teachers have not been integrally involved in either the research forming the basis for reform mandates or in the design and implementation of the reform program itself. Staff development materials are usually designed and staff development activities are usually conducted by teacher educators, in consultation with educational researchers. Contact between teachers and researchers has all too often been limited to teachers' preservice education or the occasional inservice program run by outside "experts." Not surprisingly, this lack of substantive interaction with researchers often leads to confusion among teachers about the plethora of reform-minded programs, doubts about the feasibility of classroom implementation ("...but that couldn't work in my school..."), and indifference. Teachers see reforms come and go (Cohen, 1987; Cuban, 1990). How can the proponents of reform answer these valid concerns of astute practitioners?

Education oriented researchers can address teacher concerns in at least two ways. One entails fostering and entering into more intensive practitioner-researcher collaborations. Work in the area of research utilization has shown that both the intensity and duration of researcher-practitioner interactions greatly affect the probability of application of a piece of research by practitioners (cf. Huberman, 1990). A second is to provide teachers with evidence of a program's effectiveness in actual classrooms with actual students. A program may work wonders in theory, but how many teachers have theoretical classrooms? Both of these approaches promote a common goal, increased teacher conviction about and participation in education reform. Unfortunately, little is known about the influence of the level of teacher empowerment in the implementation of innovative mathematics programs. The work of Carpenter, Fennema, Peterson, Chiang, and Loef (1989), Cobb et al. (1991), Lane (1991), and Stein, Grover, and Silver (1991) represent recent movement in this direction.

The Thinking Mathematics project attempted to increase teacher conviction, by engaging in intense practitioner-researcher collaboration, and by providing relevant information on student learning outcomes as a result of implementing the project. Empowerment of both teachers and students was hypothesized to result because teachers were involved in all phases of development and in dissemination[3], and attempts were made to assess the impact of the project on the students involved.

The model for practitioner-researcher collaboration that has evolved out of the Thinking Mathematics project has itself been the focus of much study and reflection (Bickel & Hattrup, 1991/1992; Hattrup & Bickel, 1992; Leinhardt & Grover, 1990). In addition, the collaboration's effects upon the various communities involved -- mathematics education researchers, the teachers, and the leaders of the professional teacher organizations--

[3]Products of the collaboration (Thinking Mathematics: Vol 1 Foundations and Thinking Mathematics: Vol. 2 Extensions) used in dissemination are described briefly in the Appendix and can be obtained from the American Federation of Teachers by writing to Dale Boatright, American Federation of Teachers, 555 New Jersey Ave. NW, Washington, DC 20001.

have been examined. Representatives from each community have identified important consequences resulting from their participation in the Thinking Mathematics project. Researchers are expanding their ideas about how to plan, conduct, and interpret research (Resnick, 1992). Teachers feel revitalized, are developing networks among their colleagues, and are developing an appreciation for their own ability to interpret research for instructional practice (Bickel & Hattrup, 1991/1992; Gill & Murakami, 1992; Grover, Gill, & Kaduce, 1991). In planning the second decade of operating its Educational Research and Dissemination (ER&D) program, the leadership of the AFT is taking into account the critical importance of teacher-researcher dialogue and the nature of the support structures necessary to effectively put research into practice (Billups, 1992).

While these are encouraging signs and might be taken to establish the potential of the collaborative model developed by the Thinking Mathematics project, the picture is incomplete without an assessment of the project's impact upon students. Students' increased mathematical understanding, ability, and enjoyment are the ultimate aims of any mathematical education reform effort. If, in the end, despite radical and promising changes observed in the principal collaborators' communities, there are no signs of progress toward meeting the project's goals for students, careful rethinking of the instructional program must occur. As Morgan aptly put it: "Children are the future. Everything we do is for them and everything that will be done will be done by them." (1988, cited in National Research Council's Everybody Counts, 1989, p. 96). How have the "real live" students who have been involved in the Thinking Mathematics project's first year of dissemination been affected by this reform-spirited effort? What has been in it for the students?

The 1990-91 Thinking Mathematics Student Assessment[4]

We were interested in learning about both affective and cognitive effects on students of the Thinking Mathematics (TM) implementation. To this end, we collected student data using three different types of assessments, each of which provided different information about the skills and interests of the students. One was a survey of student attitudes towards mathematics, another was standardized achievement test score data, and the third assessment instrument was a problem solving test. In addition, we obtained data from pilot site teachers about their perceptions of the influence their participation and their students' participation in the Thinking Mathematics project had upon their students. Results from each of these data sources will be described and discussed in turn.

It is important to note that these assessments were not designed to, and therefore do not, constitute a formal program evaluation of Thinking Mathematics, and they cannot be considered as such. The goal of the student assessment was rather to provide timely and relevant preliminary information about Thinking Mathematics' impact on students, by triangulating from multiple data sources. Each individual assessment provides only one perspective, and there are limitations associated with each. Taken together, however, they provide us with a richer understanding of how Thinking Mathematics impacted upon students and teachers on the road to empowerment.

Results and Discussion

Data was collected from students and teachers at five sites across the country (San Francisco, CA, Albuquerque, NM, Anderson, IN, Gary, IN, and Hammond, IN). There were approximately 65 classes, from grades K-5 (plus one 7th/8th LD class), in which teachers were voluntarily implementing the Thinking Mathematics approach during the 1990-91 school year. Since participation in the project's assessments was also voluntary, there are varying numbers of teachers and students whose data

[4]See Hojnacki and Grover (1991) for a detailed account of the results of the assessments addressed in this section of the paper.

contributed to each of the four assessments. Details concerning each assessment are provided in the following subsections.

1. Teachers' Views of Student Progress with Thinking Mathematics

Two teacher self-report instruments, a midyear survey and an end-of-year evaluation, were developed in order to obtain TM teachers' perspectives on the project's impact on themselves and on their students. These instruments are self-reports of teachers' experiences with Thinking Mathematics in their own classrooms. While we do not have independent observations to compare to the self-reported observations, the information gleaned from the teachers reveals common themes in their subjective impressions. The teachers' perceptions of the changes that they made in their instructional practice set a context for their perceptions of changes in students, and are reported here for that purpose.

A. Midyear survey. A six-item survey was distributed to all pilot site teachers in December 1990, toward the end of their first semester of training and classroom implementation of TM (Gill, 1990). Highlighted here are some of the views expressed by the 64 pilot teachers in that survey. What was quite clear was that the teachers reported their instructional practices to be changing. For example, a majority of pilot teachers surveyed reported that Thinking Mathematics had caused them to make adjustments to their teaching timelines (90%), had changed their use of textbooks (with some teachers dropping them completely) (80%), and had altered their grading and/or assessment practices (65%). In response to the question of "What pleases you most, at this point, about TM?", a majority of the responses (59%) were related to student factors: enhanced learning, increased motivation, and greater enjoyment in doing mathematics. For example, one 2nd grade teacher wrote: "It's exciting to see the "light bulb" go on. It takes a long time to teach, but once they have the basics, Thinking Math is so powerful. The students feel successful and powerful." The other 40% of the responses mentioned factors such as specific aspects of the TM approach (12%), aspects of TM supporting teacher professional development (10%) and collegial sharing (10%), or combinations of the above factors (8%).

B. End-of-year evaluations. Another evaluation instrument was administered to the pilot teachers toward the end of their second semester of involvement in TM (May 1991), in order to gain relevant follow-up information on teachers' views of Thinking Mathematics program and the dissemination process. Teachers received these end-of-year evaluations and responded to them a week prior to a final group "debriefing" meeting of the TM teachers at each of the pilot sites. These end-of-year evaluation sessions were videotaped at three sites and audiotaped at a fourth site. No data on the debriefing session was available from the fifth site, since it was neither audiotaped or videotaped. Approximately 15 teachers who participated in the group sessions did not individually fill out evaluation forms. However, 49 completed evaluation forms were received from all five sites.

Pilot teachers noted both direct and indirect benefits to students through their participation in Thinking Mathematics. Direct affective benefits mentioned were improved student self-confidence and self-esteem (e.g., through the encouragement to produce -- and accept -- multiple solutions to problems), and that students learned to value and respect each other's opinions. There were also direct cognitive benefits to the students perceived by the teachers, such as deeper student understanding of mathematics and greater number sense.

TM teachers viewed their own professional development and improved instruction/pedagogy as constituting indirect benefits to their students. A common theme expressed in the evaluation sessions was that their own enthusiasm and confidence about mathematics was conveyed to their students. Also, they found themselves spending more time on mathematics instruction by integrating math throughout the curriculum. These are positive indications of teacher empowerment via Thinking Mathematics.

But the teachers also expressed some frustrations about their year-long implementation of Thinking Mathematics. One was their feeling of not having enough time -- time to plan, time to work through concepts as they would have liked to, time to prepare for substitute teachers while they

were having TM group meetings (one day a month of release time after the initial five days of training), and so forth. They were also concerned with "covering the curriculum" by the end of the year, which created a conflict with the goal of allowing students sufficient time to develop deep understanding of mathematical concepts. Finally, many pilot site teachers worried about whether implementing Thinking Mathematics would place their students at a disadvantage relative to other students, in terms of their performance on standardized tests, which are timed.

This concern about test scores is by no means unique to the Thinking Mathematics project; the press for scores is felt in many other contexts nationwide (cf. Mitchell, 1992; Smith, 1991). To study if this concern had any basis, we collected standardized achievement test score data on the students involved in Thinking Mathematics in 1990-91. The data were from those national or state tests already being administered at the pilot sites. We also collected two other kinds of student data: (1) student attitudes toward mathematics (as measured by responses to the project's Attitude Survey); and (2) student problem solving abilities (as measured by performance on the Wood-Cobb Problem Solving Tests, Grades One, Two, and Three). Results and discussion of our analyses of these three data sources follows.

2. Student Standardized Achievement Test Performance

Although an increasing number of teachers hold concepts of educational attainment that they believe are not adequately captured by state-mandated achievement tests, they still feel extreme pressure from district administrators and the general public to raise scores. The project's primary goal in looking at standardized achievement test scores was to address concerns that student scores might be negatively affected by exposure to the Thinking Mathematics (TM) approach. To this end, we collected standardized test scores from a total of 1713 students in 87 classes, in Grades K-5. Fifty-one of these classes were pilot testing the Thinking Mathematics approach; 36 classes were not. Three different standardized tests (i.e., California Achievement Test - CAT, the Indiana State Test of Educational Progress - ISTEP, and the Comprehensive Test of Basic Skills - CTBS) were administered to students in TM schools.

The non-project classes cannot be considered as control groups in the usual research sense. Since teachers had voluntarily participated in Thinking Mathematics training and classroom implementation, there was not random assignment of classes to TM or non-TM conditions. No attempts were made to control the instructional formats of teachers in non-TM classes to be sure that those teachers did not employ any of the TM principles, and no attempts were made to standardize the degree of implementation of TM in project classrooms (in keeping with the voluntary philosophy of ER&D, of which TM was a part). Because of this lack of true control conditions for evaluating the impact of TM on students, formal tests of statistical significance and definitive cause/effect conclusions are inappropriate. However, some preliminary conclusions are suggested by the available data.

Figure 1 displays the mean test scores (as measured by median national percentiles [MDNPs]) for Grades 1 through 5 on the Computational subtests for students in TM classes compared to students not in TM classes. Except for Grade 1, the differences in Computational scores between TM and non-TM classes are relatively small. The non-TM mean computational score is higher than the TM mean for Grades 2 and 3, but the reverse is observed for Grades 4 and 5. The difference between the TM and non-TM classes' computational means is much greater for Grade 1, however, with the TM mean being 25% greater than the non-TM mean.

Insert Figure 1 about here

The mean test scores for the Concepts and Applications subtests for Grades 1 through 5 are shown in Figure 2. In all grades, the TM classes' mean scores are equal to (for Grade 2) or higher than (for Grades 1, 3, 4 and 5) the non-TM classes' mean scores (again, as measured by MDNPs).

Insert Figure 2 about here

Thus, the standardized test data enables us to conclude that TM students are doing at least as well or better than their non-TM peers.

These data are consistent with the results reported by Schifter and Simon (1991) when they compared performance on standardized tests of students prior to and following their enrollment in classes of teachers who had participated in the Educational Leaders in Mathematics (ELM) project. The teachers in ELM implemented a constructivist-oriented approach to teaching, an approach consistent with the NCTM standards. No significant differences were found in the standardized test scores of approximately 300 elementary and 300 secondary students involved in the study. Schifter and Simon (1991) report the following:

> Yet standardized test scores did not change. This result should help allay concerns that greater attention to understanding and problem solving, particularly considering the additional time allotted to conceptual exploration, will lead to a decline in computational skill. The related concern that instructional changes of this magnitude will result in lower test scores for the first year or two, as teachers learn the ropes, has also been expressed. However, these test results indicate that even during the initial change process, computational skill is not necessarily sacrificed. (p. 48)

Using student data collected from multiple choice tests as the only measure of student mathematical achievement is universally decried as being inadequate and misleading (cf. Mitchell, 1992; NCTM, 1989). Our purpose in collecting these data was to contribute to a broad sketch of the performance of TM students, to be used to supplement the data from the project's attitude survey and the problem solving tests. The current standardized test data cannot be used to establish that the TM approach enhances or diminishes student performance on these standardized tests. From this information, it appears that the performance is at least comparable to or better than that of students in classes which were not part of the pilot implementation. These results, as with Schifter and Simon's (1991), should help allay concerns about declines in computational skill.

3. Student Attitudes towards Mathematics

The affective aspects of student participation in Thinking Mathematics were directly assessed by the project-wide administration of an attitude inventory. The "Students' Attitudes Toward Mathematics" Survey consisted of 24 items from three scales: a) a *Mathematical Confidence* scale, reflecting how confident students felt about their mathematical abilities; b) a *Mathematical Motivation* scale, reflecting how motivated students were to do math (in a classroom setting); and c) a *General Attitudes towards Mathematics* scale, which reflected students' more general attitudes about the subject and its learnability. Each scale consisted of 8 items. Half of the survey items were phrased positively (e.g., Item #1: "I am sure that I can learn math."), and half were phrased negatively (e.g., Item #4: "I forget most of the things that I learn in math.") Most of these items were drawn from several existing attitude scales (Fennema-Sherman Mathematics Attitudes Scales; Second International Mathematics Study; International Study of Achievement in Mathematics; Mathematics Attitude Inventory). Additional items were constructed and added to the set to equalize the number of positively and negatively phrased items in each scale. There were also two open-ended questions on the survey: a) "What do you like most about math class?", and b) "What bothers you the most about math class?". Pilot testing of the Attitude Survey items and format was conducted in September 1990. On the basis of this pilot testing, we restricted the administration of the survey to those students in 2nd grade or above. Kindergarten and 1st grade students did not have sufficient understanding of the concepts being addressed in the inventory, making the validity and reliability of their responses questionable.

Approximately 540 Thinking Mathematics students participated in the attitude survey administered in May 1991. There were three different test formats (a 24-item Yes/No format, a 24-item Yes/Maybe/No format, and a 22-item format Yes/Maybe/No format). The "maybe" response was included as a choice on two of the test formats in order to see how student response patterns changed when there was a "maybe" option. The 22-item format was an earlier version of the 24-item Attitude Survey and was

administered at one of the TM pilot sites. The two 24-item versions were identical except for inclusion of the "maybe" response. Preliminary analyses of all three test formats indicated that the response patterns on the Yes/Maybe/No format surveys were not markedly different from the response patterns of the Yes/No format survey. Therefore, detailed analyses were conducted only upon the Yes/No format of the survey (n=250). The scores reported reflect the percentages of students giving responses that indicated positive attitudes about math, regardless of how the item was worded. Throughout this discussion, we use the phrase "positive responses" to mean responses that indicate a positive attitude toward mathematics, not necessarily a "Yes" response.

Figure 3 displays the mean percentages of positive responses for each scale, as well as breakdowns by grade for each scale. Overall, the mean percentage of positive responses across all scales and grades was 79%, indicating that students did indeed have very favorable attitudes towards mathematics. Students had the most positive responses to the General Attitudes toward Mathematics (General to Mathematics) scale items and the least positive responses to the Motivation scale items. Grade trends indicate that the 4th graders typically held less positive attitudes towards mathematics than the younger grade students did.

Insert Figure 3 about here

The item with the highest percentage of positive responses (97%) was from the Confidence scale, Item #1: *I am sure that I can learn math.* More than 90% of the students gave a "No" response to two other items, from the General to Mathematics scale (Item #14: *I dislike everything about math*, and Item #15: *Only a few people can learn math*), indicating a highly positive attitude towards mathematics. Illustrative of the more ambivalent ratings students accorded to Motivation items overall, two of the three lowest items were from the Motivation scale (Item #10: *I like easy math problems the best* -- 57% "No"; and Item #20: *Sometimes I work more math problems than are assigned in class* -- 53% "Yes"). The third item representing a somewhat negative attitude was from the

Confidence scale (Item #16: *I often think "I can't do it" when a math problem seems hard* -- 44% "Yes").

One possible explanation for these results[5] may be that the longer the exposure to negative experiences, the more difficult it is to change attitude. Fourth graders may have had three or four years (K to 3rd grade) of negative experiences whereas second graders have had only one or two years (K and 1st grade). One year of the TM approach apparently has less impact on the more ingrained attitudes of the older students.

Interestingly, our findings concerning the mathematical attitudes of young elementary school children prefigure the attitudes of the 8th and 12th grade students taking part in the SIMS study (McKnight et al., 1987). McKnight et al. (1987) characterized their older students as perceiving themselves as capable in mathematics but with a limited commitment to it. Less than 50% of their 8th grade students believed that mathematics was enjoyable or spent a lot of their own time doing mathematics. However, these students viewed mathematics overall as a largely fixed, rule-oriented body of knowledge to be acquired through memorizing and learning to apply rules. The Thinking Mathematics approach encourages students to develop a very different conception of mathematics, one that is much less dogmatic and that values students' own solution strategies as much as, if not more than, the traditional algorithms. Although we did not similarly explore TM students beliefs about the nature of mathematics, we predict that they would not share these views of their SIMS' peers.

Student responses to the two open-ended questions on the survey largely confirmed findings from the Yes/No survey items. The single most common response to the question "What do you like most about math class?" was that math is fun. The most frequent response to the question "What bothers you most about math class?" was "Nothing bothers me about math class".

The available data on student attitudes are limited in a number of important ways. We do not have any comparative data, either from the

[5] This explanation was suggested by a third grade teacher-colleague who is active in TM dissemination and implementation efforts.

students' attitudes about mathematics prior to their experiences with Thinking Mathematics or from the TM students' peers in non-TM classrooms. Although it is certainly a positive finding that students enjoy mathematics, we cannot conclude that positive affect necessarily translates into improved skills and abilities. Indeed, several studies that have examined mathematics attitudes/skills relationships have discovered little covariation (cf. McKnight et al., 1987; Stevenson, Lummis, Lee, & Stigler, 1990). Compared to their Asian counterparts, for example, U.S. students have better attitudes but less ability (the latter as measured on international achievement tests). U.S. students and their parents believe that the (U.S.) students are good at math, although international comparisons point to other conclusions (McKnight et al., 1987; Stevenson et al., 1990).

4. Student Problem Solving Performance

In addition to collecting information on students' attitudes and students' mathematical achievement (as measured by standardized multiple choice tests), we wanted to obtain information about Thinking Mathematics students' problem solving abilities, since problem solving is a primary instructional focus of the TM approach (see Appendix A). Three problem solving instruments for Grades One, Two, and Three designed by Wood and Cobb (1989)[6] were administered to Thinking Mathematics students to assess their problem solving performance prior to and following a year of TM instruction. We were interested not only in the change in their pretest to posttest accuracy, but also in the kinds of errors that students made. Both accuracy and error information have potential for diagnostic use in the classroom.

The 25-item Grade One test was administered to students in ten first grade classes. The 33-item Grade Two test was administered to students in ten second grade classes. The 33-item Grade Three test was

[6] In developing these three problem-solving tests, Wood and Cobb's goals were to "develop paper-and-pencil items that assess children's understanding [of arithmetic and that] offer a useful alternative to the traditional 'skills' approach to mathematics assessment" (p. 13). All the items are open-ended. Collectively, they assess children's understanding of number concepts, place value, and the four arithmetical operations.

administered to students in eleven third grade classes, nine fourth grade classes, and one 7th/8th grade class for learning disabled students. Data were obtained from 252 students in a random sampling design, representing approximately 25% of each class. Pretests were administered in September 1990, and posttests in May 1991. More students took the posttest than the pretest, since some pilot site teachers did not administer pretests. Data from those students for whom both pretest and posttest scores were available (N=158) was used for accuracy analyses and pre/posttest comparisons. Data from all 252 students, however, entered into error analyses.

A. Accuracy, Common Errors, and Idiosyncratic Errors. Answers to the problem solving items were categorized as either being correct answers or errors; the 5032 errors were further categorized as being either common or idiosyncratic ones. For each item (pretest and posttest), specific errors produced by three or more students were noted; these were categorized as being common errors, and were the focus of subsequent analysis. Our rationale for this focus was that these were predictable errors that teachers were most likely to see again. Errors that were made by only one or two students were categorized as idiosyncratic ones. [We encouraged teachers to explore these types of errors more fully with the individual students who made them, but no further analysis was performed on them collectively.]

The mean percentages of responses in each of three categories are shown in Figure 4 and Table 1.[7] Posttest accuracy rates were much higher than pretest rates for all three tests on paired one-tailed t-tests (Gr. 1: $t(37)=+11.37$; Gr. 2: $t(44)=+19.65$; Gr. 3: $t(74)=+11.12$, all three at $p < .0001$), with gains ranging from 25% (on the Grade Three test) to 45% (on the Grades One and Two tests). Insofar as could be measured by these particular tests developed by Wood and Cobb (1989), Thinking Mathematics students were definitely learning problem solving skills. As is the case for the attitude survey and standardized achievement test

[7] We provided teachers with more detailed information for both accuracy and error patterns, to aid them in instructional planning; this data is reported in Hojnacki and Grover (1991).

scores, however, we did not have suitable comparisons. The items on this test were not standardized or norm-referenced.

Insert Figure 4 about here

Insert Table 1 about here

As can be seen in Table 1, errors on the Grade Three test were split about equally, both on the pretest and the posttest, between those that were common (Pre: 28%; Post: 14%) and those that were idiosyncratic (Pre: 27%; Post: 14%). A different pattern emerged for errors on the Grade One test. The percentage of errors was split about equally for the pretest, but idiosyncratic errors accounted for nearly twice as many errors as common errors did on the posttest. The reverse was true on the Grade Two test. On the Grade Two test, the percentage of errors was split about equally for the posttest, but on the pretest, there were twice as many idiosyncratic as common errors. At this point of our analyses, we cannot offer a compelling explanation for these patterns.

We examined pre- and posttest differences in the range of answers -- the spread from the smallest incorrect answer to the largest one -- produced on the three tests. This provides a measure of the extent of random guessing. For example, on Item #1 of the Grade One pretest, the answers ranged from "1" to "13" (the correct answer was "10"), a range of 12. The mean range of answers on each of the three problem solving tests are shown in Table 2. The mean ranges for the Grades Two and Three are higher than the mean ranges for the Grade One test; this is probably due to the inclusion of more two- and three-digit numbers on the tests and thus the greater opportunity for answers to vary widely. Paired two-tailed t-tests conducted on these data indicate that the range of answers narrowed significantly from the pretest to the posttest on the Grades One and Two tests (Gr. 1: $t(24)=2.45$, $p < .05$; Gr. 2: $t(30)=2.25$, $p< .05$). There were no significant differences observed on the Grade Three test. Students in the

two younger grades seemed to be honing in on the right answer by posttest time, even if they did not get it correct.

Insert Table 2 about here

B. Types of Common Errors. The ideal situation for diagnosing student errors is to interview students at the time when they produce the errors -- in the course of problem solving. Due to the scope and purpose of the assessment, we were not able to do this; only the errors were available for analysis.

We examined the common errors for each item, and inferred what strategy students might have been using in order to arrive at these errors. These were grouped into five categories of common errors:

1. *Computational Errors* were answers that were ±1 digit (and, in a few cases, ±2 digits) away from the correct answer. The problem solving strategy was probably correct, but the student miscounted and, thus, got an incorrect answer.

2. *Operational Errors* indicated the use of an incorrect arithmetical operation (e.g., subtraction instead of addition, addition instead of multiplication, etc.). These kinds of errors probably indicate that the student either does not understand what the signs (+, -, x, ÷) mean or does not understand why a particular operation is required.

3. *Conceptual/Procedural Errors* include a broader range of errors. These errors included those where students appeared to be using an appropriate procedure incorrectly (e.g., "borrowing" from the tens column without subtracting a ten), which could be a good indication that the student making such a procedural error does not understand the conceptual rationale for the procedure. Also included in this category were those errors that dealt with misunderstandings of place value.

4. *Problem Misunderstanding Errors* also include a broad range of errors. This category included all those errors which seemed to indicate that the student did not interpret the problem information correctly (e.g., only doing the first step of a two-step problem; not ignoring the extraneous information sometimes accompanying problems). The most frequent error of this type, which was especially common on the Grades One and Two tests, was to reiterate one of the quantities mentioned in the problem statement.

5. *Indeterminate Strategy Errors* were errors for which no strategy could easily be inferred. This does not mean that students were just guessing, however. In fact, because at least three students had to produce a specific answer before it was classified as a common and not an idiosyncratic one, random guessing is even less likely. However, there was insufficient information to decide why students made these errors.

Breakdowns of the percentages of common errors categorized into the five types for each grade's test are provided in Figure 5. The percentages of common errors out of total responses (including correct answers and idiosyncratic errors) for each grade's test was displayed in Table 1 and are summarized below.

Insert Figure 5 about here

The most salient difference between the pre- and post-tests is, of course, the increase in the percentage of correct answers, which we have already discussed. Since there were fewer errors produced on the posttest, there were corresponding decreases in the percentages of each type of common error. On the Grade One test, the five types of common errors represent fully 38% of all answers on the pretest, but only 8% on the posttest. On the Grade Two test, the decrease was from 26% to 12%; on the Grade Three test, the decrease was from 28% to 14%.

The single consistent change across all three tests is that the percentage of *Indeterminate Strategy Errors* decreased the most from pre- to post-test (decreases were 21%, 26%, and 10%, respectively, on the Grades One, Two, and Three tests). *Computational Errors* were more frequent, especially on the posttests, for the two younger grades. On the Grade Three test, the two main error types (pre and post) were *Operational Errors* and *Conceptual/Procedural Errors*. This corresponds to the absolute increase in number of operations, concepts, and procedures students are asked to understand by Grade Three.

C. Instructional Implications of Problem Solving Test Results. The individual teacher is the best qualified to determine what the instructional implications of these results are for his/her group of students. However, in the spirit of the reform efforts that advocate that assessments should inform instructional practice, we shared some of our own ideas with our pilot site teachers in the report of the student assessment (Hojnacki & Grover, 1991). We encouraged the teachers to discuss the assessment as a whole and in particular the instructional implications of the problem solving test results at their Thinking Mathematics professional development sessions.

First, the classification of common errors that emerged from these data could provide a framework from which teachers can develop questioning techniques and strategies to probe more fully the thinking and reasoning patterns behind students' solution processes. In particular, one of the most frequent error types on the Grades One and Two tests in the Problem Misunderstanding category was the reiteration of one of the quantities mentioned in the problem statement. Why do students do this? Are they focusing erroneously on some language cues? Are they just guessing? Why do they believe the correct solution would be provided in the problem?

Secondly, the value of having students articulate their reasoning seems to be reinforced by these results. Since Problem Misunderstanding errors were most common on the Grades One and Two tests, there may be more need to spend time with language and talking about the meaning of

the problem in a variety of ways (which is in fact one of the research-based principles of TM). The same remedy might also lower the incidence of Operational and Conceptual/Procedural errors on the Grade Three test, by getting students into the habit of talking through problems and procedures and understanding what information is being required. Also, additional probing by teachers, in the form of requesting students to describe and justify their solutions in a class setting, might shed some light on the Indeterminate Strategies solutions. It is possible that the common decrease observed for all grades in the percentage of Indeterminate Strategies at the posttest might be partially attributable to the yearlong effects of emphasizing students' making sense of their answers. An additional classification of errors might emerge from a teacher's probing and suggest particular instructional strategies to address those errors in thinking. Or, it may be that the errors would fall into one of the already established categories and suggest a greater preponderance of misconceptions in a particular area than these data indicate.

D. Summary of Problem Solving Test Results. For all grades, accuracy increased dramatically and the range of errors narrowed from pre to post test. Error analyses indicated that Computational errors were more frequent on the Grades One and Two tests than on the Grade Three test (pre and post), while Operational and Conceptual/Procedural errors were the most common type observed on the Grade Three test. The percentage of idiosyncratic errors decreased from pre to post test for all grade levels. Teacher knowledge of these error patterns can be useful in classroom diagnosis and instructional planning.

Conclusions

How have the analyses just described informed us about student learning outcomes in the Thinking Mathematics program? What evidence of student empowerment via teacher empowerment is there? The available data on teachers' experiences with Thinking Mathematics (Gill & Murakami, 1992; Billups & Rauth, 1992) supports the view that teachers reported being changed in empowering ways. The project's teacher self-report data show that these teachers perceived empowering changes in

their students, as well. Performance on the Wood-Cobb tests indicated notable improvements in student problem solving ability. Positive student attitudes towards mathematics were revealed by the project's attitude survey; students' lower motivation scores, however, suggest that they are not yet ready to match their efforts with their generally positive regard for mathematics. TM students performed as well as or better than their non-TM peers on both the Computational and the Concepts and Applications subsections of the standardized tests. Although the results reported from the 1990-91 Thinking Mathematics student assessment are preliminary, there are multiple indications that student learning and attitudes were enhanced by their participation in the program. The collaborative model developed by the Thinking Mathematics project seems to be paying off for both teachers and students.

Along the way, we have discovered many difficulties inherent in attempting to assess student change as a function of instructional change. From a research perspective, these difficulties were compounded because of the completely voluntary, non-threatening, and non-judgmental nature of the instructional change program. With Thinking Mathematics currently in its second year of implementation, another student assessment is underway. The lack of control over the extent of TM implementation is unavoidable. However, we will be in a better position to evaluate change in student attitudes towards mathematics by pre- and post-testing. Also, we will be able to compare problem solving ability over two years. The additional data collected should allow us to more fully address questions concerning student outcomes.

References

Bickel, W. E., & Hattrup, R. A. (1991/1992). A case study of institutional collaboration to enhance knowledge use: Restructuring practitioner-researcher dialogue in education. Knowledge and Policy: The International Journal of Knowledge Transfer, 4(4), 56-78.

Billups, L. (1992, April). The three Rs: Research rounded reformers. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Billups, L., & Rauth, M. (1992). Epilogue. In G. Leinhardt, R. Putnam, & R. A. Hattrup (Eds.), Analysis of arithmetic for mathematics teaching (pp. 431-439). Hillsdale, NJ: Lawrence Erlbaum Associates.

Bodenhausen, J., Denhart, N., Gill, A., Kaduce, M., Miller, M., Grover, B., Resnick, L., Leinhardt, G., Bill, V. Rauth, M., & Billups, L. (in press). Thinking Mathematics Vol. 1: Foundations & Vol. 1T: Training Modules. Washington, DC: American Federation of Teachers.

Carpenter, T. C., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematical thinking in classroom teaching: An experimental study. American Educational Research Journal, 26(4), 499-531.

Cobb, P., Wood, T., Yackel, E., Nicholls, J., Wheatley, G., Trigatti, B., & Perlwitz, M. (1991). Assessment of a problem-centered second-grade mathematics project. Journal for Research in Mathematics Education, 22(1), 3-29.

Cohen, D. (1987, May) Teaching practice: Plus que ca change. Paper presented at the Benton Center for Curriculum and Instruction, University of Chicago, Chicago, IL.

Cohen, D. K., & Ball, D. L. (1990). Policy and practice: An overview. Educational Evaluation and Policy Analysis, 12 (3), 347-353.

Cuban, L. (1990). Reforming again, again, and again. Educational Researcher, 19(1), 3-13.

Gill, A. J. (1990, December). [Survey of Thinking Mathematics pilot teachers: Results]. Unpublished raw data.

Gill, A., & Grover, B. (Eds.). (in press). Thinking Mathematics Vol. 2: Extensions & Vol. 2T: Training Modules. Washington, DC: American Federation of Teachers.

Gill, A. J., & Murakami, L. (1992, April). Professional growth among teachers: Teachers as agents of change. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Grover, B. W. (1991). The teacher assessment dilemma: What is versus what ought to be! Journal of Personnel Evaluation in Education, 5, 103-119.

Grover, B. W., Gill, A. J., & Kaduce, M. (1991, April). Teacher and researcher collaborations: Implementing the NCTM Standards. Paper presented at the research presession of the 69th annual meeting of the National Council of Teachers of Mathematics, New Orleans, LA.

Hattrup, R. A., & Bickel, W. E. (1992, April). Lessons learned: Design of researcher-practitioner collaborations. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Hojnacki, S. K., & Grover, B. W. (1991). AFT/LRDC Thinking Mathematics Project: 1990-91 Student Assessment Report. Pittsburgh, PA: Learning Research & Development Center, University of Pittsburgh.

Huberman, M. (1990). Linkage between researchers and practitioners: A qualitative study. American Educational Research Journal, 27(2), 363-391.

Jacullo-Noto, J. (1986). Interactive research and development - Partners in craft. In A. Lieberman (Ed.), Rethinking school improvement: Research, craft, and concept (pp. 176-190). New York: Teachers College Press.

Lane, S. (1991, April). The conceptual framework for the development of a mathematics assessment instrument for QUASAR. Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Leinhardt, G., & Grover, B. W. (1990, April). Research and teacher-based knowledge: Synthesis through interpretive dialog. Paper presented at the annual meeting of the American Educational Research Association, Boston, MA.

Leinhardt, G., Putnam, R., & Hattrup, R. A. (1992). Analysis of arithmetic for mathematics teaching. Hillsdale, NJ: Erlbaum.

Lieberman, A. (Ed.). (1986). Rethinking school improvement: Research, craft, and concept. New York: Teachers College Press.

Lieberman, A., & Miller, L. (1984). Teachers, their world, and their work: Implications for school improvement. Alexandria, VA: Association for Supervision and Curriculum Development.

Mathematical Sciences Education Board. (1991). For good measure: Principles and goals for mathematics assessment. Report of the National Summit on Mathematics Assessment. Washington, DC: National Academy Press.

McKnight, C. C., Crosswhite, F. J., Dossey, J. A., Kifer, E., Swafford, J. O., Travers, K. J., & Cooney, T. J. (1987). The underachieving curriculum: Assessing school mathematics from an international perspective. Champaign, IL: Stipes Publishing Co.

McLaughlin, M. W. (1990). The Rand Change Agent Study revisited: Macro perspectives and micro realities. Educational Researcher, 19(9), 11-16.

Mitchell, R. (1992). Testing for learning: How new approaches to evaluation can improve American schools. New York: Free Press.

National Board for Professional Teaching Standards. (1989). Toward high and rigorous standards for the teaching profession: Initial policies and perspectives of the national Board for Professional Teaching Standards. Detroit, MI: National Board for Professional Teaching Standards.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

Passow, A. H. (1986). Beyond the commission reports: Toward meaningful school improvement. In A. Lieberman (Ed.), Rethinking school improvement: Research, craft, and concept (pp. 206-218). New York: Teachers College Press.

Resnick, L. B. (1992, April). Changing directions for research and researchers. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Romberg, T.A. (1986). Elementary school mathematics: The IGE legacy. Education, 107(1), 29-34.

Schifter, D. & Simon, M. (1991). Towards a constructivist perspective: The impact of a mathematics teacher inservice program on students. In R. G. Underhill (Ed.), Proceedings of the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education: Vol. 1 (pp. 43-49). Blacksburg, VA: Christiansburg Printing Company.

Schwartz, J. (1986). Teacher directed in-service: A model that works. In A. Lieberman (Ed.), Rethinking school improvement: Research, craft, and concept (pp. 191-205). New York: Teachers College Press.

Shanker, A. (1985). The revolution that's long overdue. Phi Delta Kappan. 66(5), 311-15.

Silberman, C. E. (1970). Crisis in the classroom: The remaking of American education. New York: Random House.

Smith, M. L. (1991). Put to the test: The effects of external testing on teachers. Educational Researcher, 20(5), 8-11.

Stein, M. K., Grover, B. W., & Silver, E. A. (1991). Changing instructional practice: Capturing the details. In R. G. Underhill (Ed.), Proceedings of the Thirteenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education: Vol.1 (pp. 36-42). Blacksburg, VA: Christiansburg Printing Company.

Stenmark, J. K. (1991). Mathematical assessment: Myths, models, good questions, and practical suggestions. Reston, VA: NCTM.

Stevenson, H. W., Lummis, M., Lee, S., & Stigler, J. W. (1990). Making the grade in mathematics: Elementary school mathematics in the United States, Taiwan, and Japan. Reston, VA: National Council of Teachers of Mathematics.

Wood, T., & Cobb, P. (1989). The development of a cognitively-based elementary school mathematics test. West Lafayette, IN: Purdue University, School Mathematics and Science Center.

Table 1

Mean Percentages of Responses per Category (Pretest and Posttest) on each Problem Solving Test

		Pretest	Posttest	Paired one-tailed t-test
Grade 1	Correct Answers	24	77	11.37*
	Common Errors	38	8	
	Idiosyncratic Errors	38	15	
Grade 2	Correct Answers	24	72	19.65*
	Common Errors	26	12	
	Idiosyncratic Errors	50	16	
Grade 3	Correct Answers	45	72	11.12*
	Common Errors	28	14	
	Idiosyncratic Errors	27	14	

*$p < .0001$

Table 2

Mean Range of Answers per Item for Problem Solving Tests

	PRETEST	POSTTEST	Paired t-test
Grade 1 Test	77.4	22.4	2.45*
Grade 2 Test[a]	286.8	127.3	2.25*
Grade 3 Test[a]	282.4	279.8	0.97

Notes.

a Two outlier items with ranges of over 1000 were removed from the analysis.

*$p < .05$

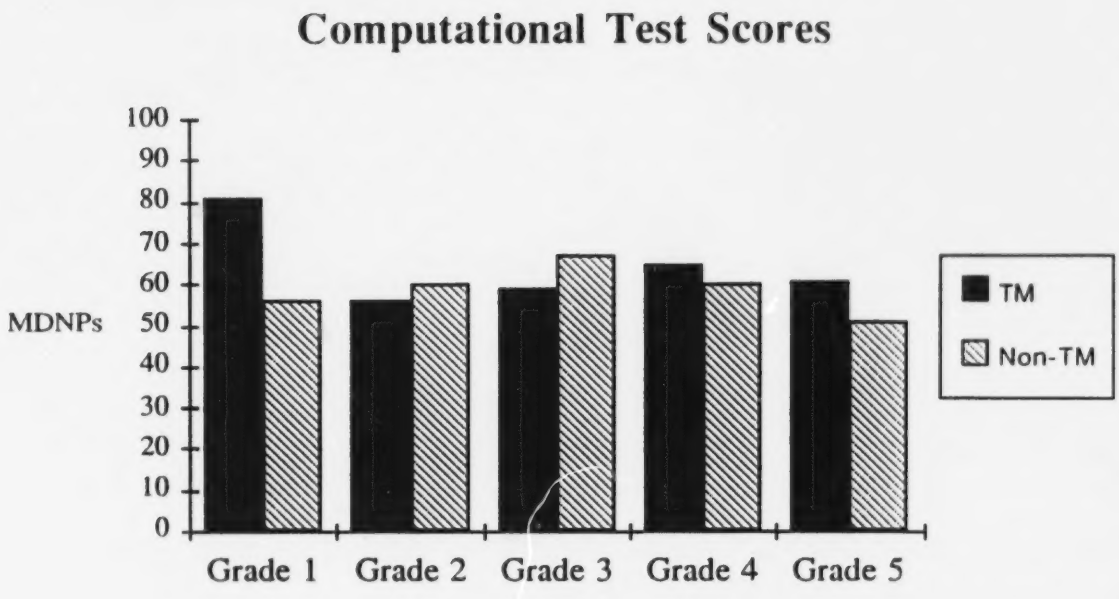

Figure 1. Mean median national percentile (MDNP) scores for Thinking Mathematics classes and non-Thinking Mathematics classes on the Computational subtests of standardized achievement tests.

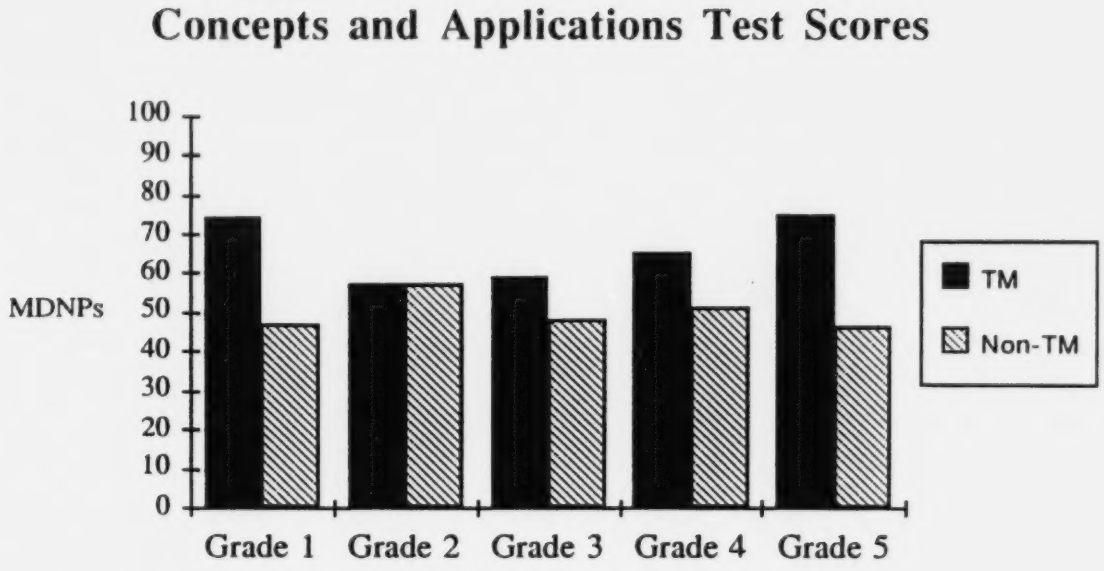

Figure 2. Mean median national percentile (MDNP) scores for Thinking Mathematics classes and non-Thinking Mathematics classes on the Concepts and Applications subtests of standardized achievement tests.

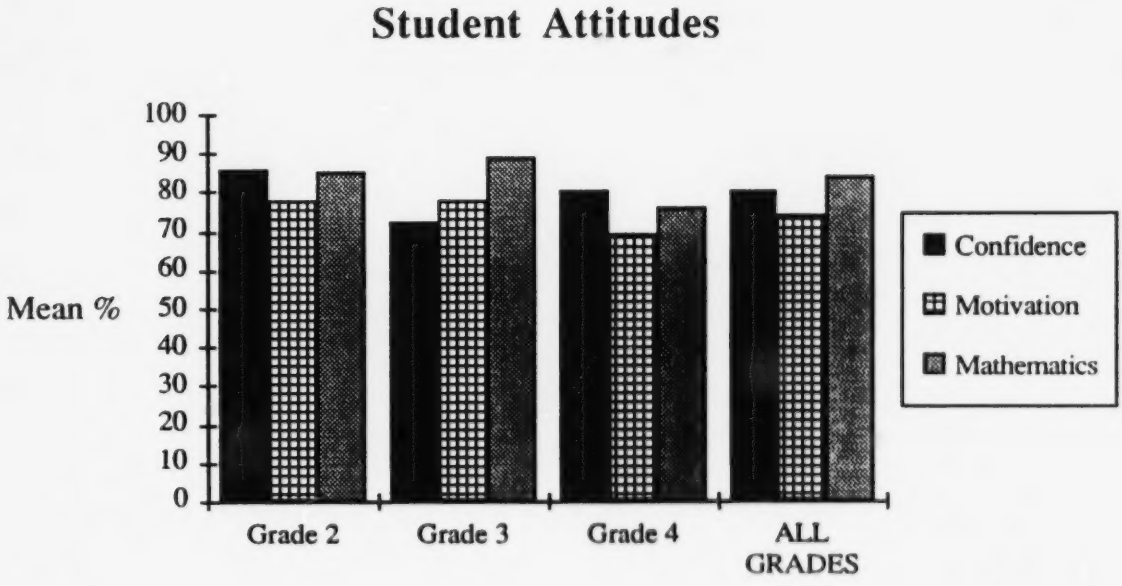

Figure 3. Mean percentages of positive responses to items on the Thinking Mathematics "Students' Attitudes Toward Mathematics" Survey, by scale and student grade level.

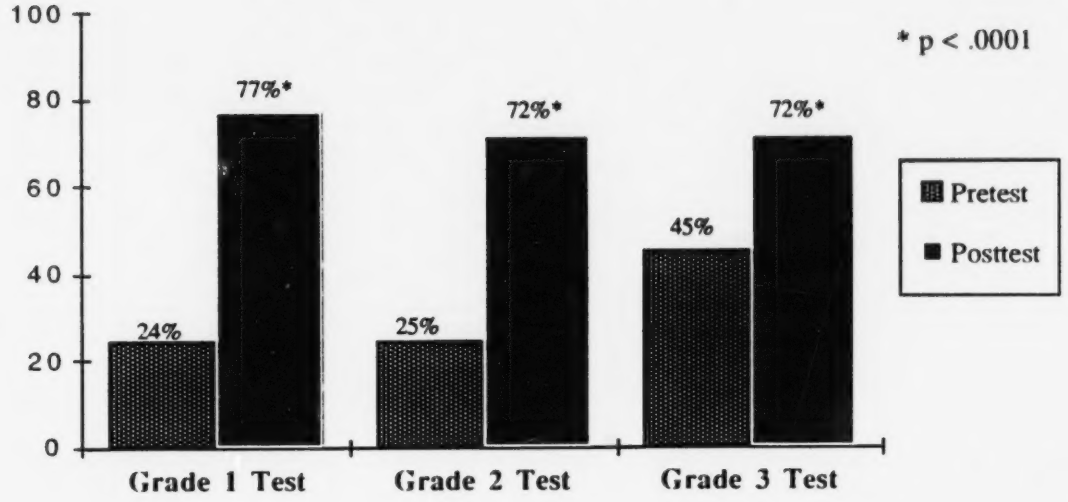

Figure 4. Mean percentages of correct responses on the Grades One, Two, and Three Wood-Cobb Problem Solving Tests (pretests and posttests).

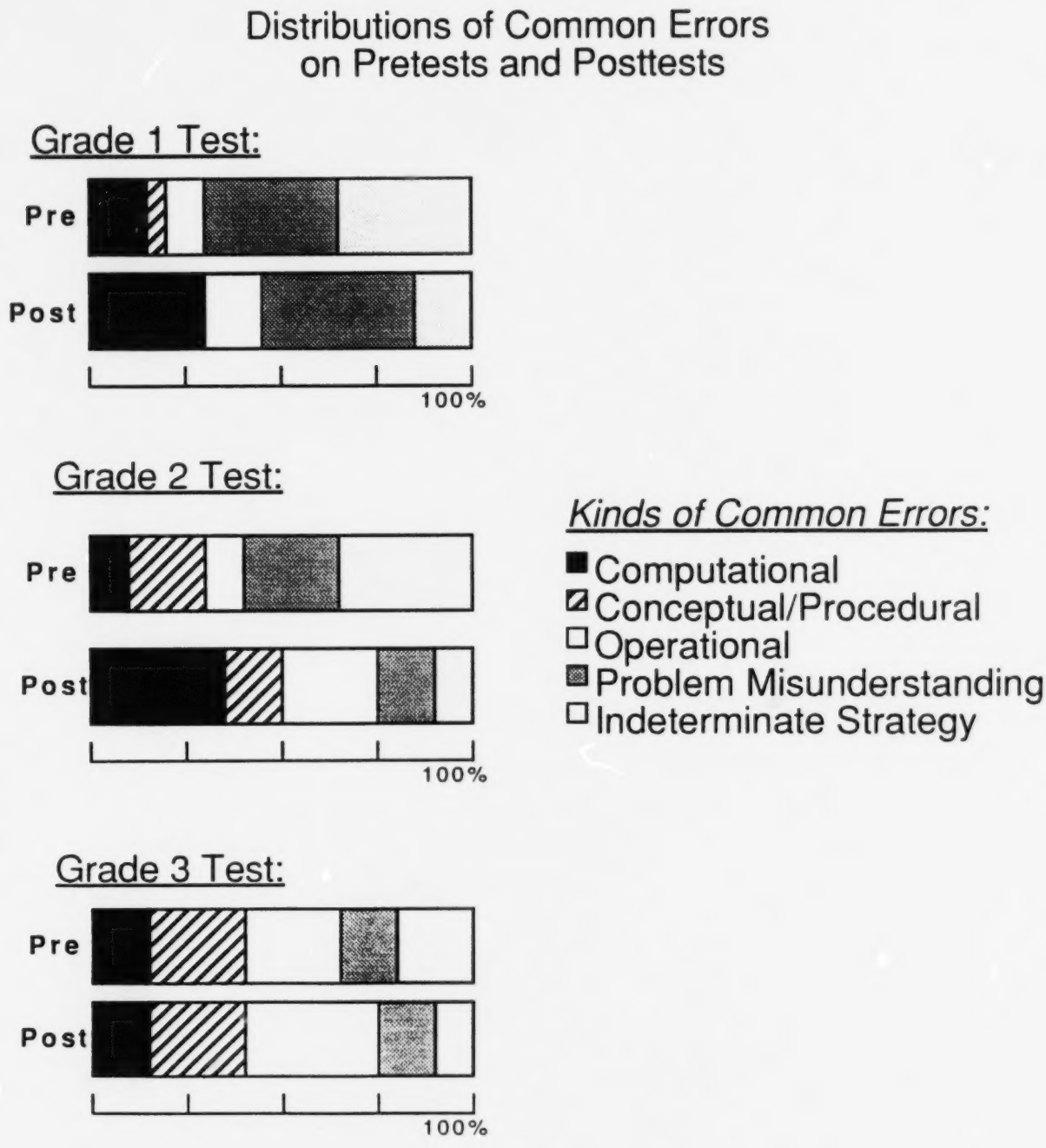

Figure 5. Percentages of different types of common errors on the Thinking Mathematics Wood-Cobb Problem Solving Tests by grade, on pretests and posttests.

Appendix A

The AFT/LRDC Thinking Mathematics Project

The Thinking Mathematics (TM) project, a 54 month project begun in 1987, is a collaborative effort of the American Federation of Teachers (AFT) and the University of Pittsburgh's Learning Research & Development Center (LRDC), sponsored by a grant from the National Science Foundation. The collaboration's primary objective is to develop more effective ways of disseminating new knowledge about mathematics instruction and learning, based upon the clinical wisdom of [expert] classroom mathematics teachers and upon current research findings.[1] It was hoped that by involving teachers (along with researchers) in all phases of the interpretation of the research, the development of professional development materials, and the dissemination process, the project would contribute toward bridging the research-practice gap.

After three years of intense teacher-researcher dialogue, informal classroom implementation and informal teacher-to-teacher dissemination, and an incredible amount of writing and rewriting, TM reached a critical phase. The first volume, Thinking Mathematics: Foundations (Bodenhausen et al., in press), which covered the subject matter topics of counting, addition, and subtraction, entered its first year of formal pilot testing in 1990-91. Teachers and students at six sites across the country -- San Francisco, CA, Albuquerque, NM, Anderson, IN, Gary, IN, Hammond, IN, and Dade County, FL -- participated. Classes ranged from kindergarten to 7th grade, including some bilingual and special education classes. The second volume, Thinking Mathematics: Vol. 2 Extensions (Gill & Grover, in press), covered the subject matter topics of multiplication and division. Pilot testing for this volume occurred during 1991-92 at five of the six sites that piloted the Volume 1 materials.

---

[1] These research findings for various topics in elementary arithmetic were summarized by prominent educational researchers for a specially commissioned volume, Analysis of arithmetic for mathematics teaching, edited by Leinhardt, Putnam, and Hattrup (1992).

Theoretical Foundation of Thinking Mathematics: Ten Principles

The first volume synthesizes the research findings which suggest a new approach to teaching mathematics that promotes the revolutionary idea that all students can learn mathematics. These research ideas suggest that teachers should embed instruction in problem solving activities, encourage students to explore, explain and justify procedures for solving problems, and should encourage discussion among the students so that they share their different approaches to solutions and articulate their reasoning in language that is understandable by their fellow students. This is not to say that children should be allowed to explore without direction in the hope that they will discover the important mathematical ideas on their own. The program of instruction is orchestrated by the teacher in an overall plan that integrates ideas to achieve particular goals. These ideas are elaborated in the project's written materials (Bodenhausen et al., in press; Gill & Grover, in press).

The following Ten Principles summarize the collaboration's collected wisdom on what characterizes good, research-based mathematics instruction:

1. Build from students' intuitive knowledge
2. Establish a strong number sense
3. Base instruction on situational story problems
4. Use manipulatives to represent problem situations
5. Require students to describe and justify their mathematical thinking
6. Accept multiple correct solutions
7. Balance conceptual and procedural learning
8. Use a variety of teaching strategies
9. Use ongoing assessment to guide instruction

10. Adjust the curricular timeline

Teacher-to-teacher collaboration and support were discovered to be increasingly important aspects of the TM dissemination process of subject matter grounded research (Grover, Gill, & Kaduce, 1991; Billups, 1992; Gill & Murakami, 1992). The AFT's ten-year-old Educational Research & Dissemination network had always utilized practitioners in both the development of research translations to be used in teacher training and in the conducting of national ER&D training. However, the extent to which the newly-trained teachers needed sustained support and follow-up training grew more apparent in TM's pilot year. The opportunity to discuss with other practitioners what they saw happening or not happening in their classrooms was essential. TM was unlike any of the other ER&D training modules in its focus on learning and instruction in a particular subject matter -- mathematics. The earlier ER&D training focused on general instructional practice, applicable to any subject matter. For example, some of the topics disseminated via ER&D concern beginning-of-year classroom management, cooperative small groups, critical thinking, and teacher praise.

As is customary with ER&D, teacher-trainers at the pilot sites participated in a week-long summer training workshop (led by the Thinking Mathematics teacher developers), and attended a three-day mid-year reunion. But recognizing the increased needs for additional assistance and teacher-to-teacher collaboration, several practices novel to ER&D were initiated during the 1990-91 pilot year of Thinking Mathematics. Two of the five teacher developers of Thinking Mathematics were released from teaching in order to provide full-time support to the pilot teachers and teacher-trainers across the country. Teachers at the pilot sites were provided with ten release days during the year: five days for initial training and one day a month for follow-up training and group meetings. Teacher-trainers were provided with an additional release day each month for preparation and administrative responsibilities. Finally, a Thinking Mathematics newsletter ("The Right Angle") was published, with the goals of keeping Thinking Mathematics teachers connected to each other and which allowed them to share classroom experiences.

During the 1990-91 TM pilot year, data were collected from the teachers about changes in their instructional practice and the impact their participation in the project had on their perceptions of themselves as teachers. Sources of data were structured interviews, written surveys, and video or audio tape recordings of discussion sessions held at their home schools with their colleagues. These data provided some insights into the sense of empowerment teachers enjoyed as a result of their participation in the collaborative project.

In addition, data were collected to provide insights into the impact on students enrolled in classes of teachers who participate in such projects. Four sources provided data on students: teacher self-report questionnaires to obtain their perceptions of the impact on their students, a student attitude survey, a problem solving assessment, and standardized test scores.